5.2 Properties of Rational Exponents and Radicals

Essential Question How can you use properties of exponents to simplify products and quotients of radicals?

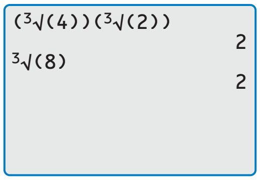
EXPLORATION 1 Reviewing Properties of Exponents

Work with a partner. Let a and b be real numbers. Use the properties of exponents to complete each statement. Then match each completed statement with the property it illustrates.

Statement	Property
a. $a^{-2} =$ ______, $a \neq 0$	**A.** Product of Powers
b. $(ab)^4 =$ ______	**B.** Power of a Power
c. $(a^3)^4 =$ ______	**C.** Power of a Product
d. $a^3 \cdot a^4 =$ ______	**D.** Negative Exponent
e. $\left(\frac{a}{b}\right)^3 =$ ______, $b \neq 0$	**E.** Zero Exponent
f. $\frac{a^6}{a^2} =$ ______, $a \neq 0$	**F.** Quotient of Powers
g. $a^0 =$ ______, $a \neq 0$	**G.** Power of a Quotient

USING TOOLS STRATEGICALLY

To be proficient in math, you need to consider the tools available to help you check your answers. For instance, the following calculator screen shows that $\sqrt[3]{4} \cdot \sqrt[3]{2}$ and $\sqrt[3]{8}$ are equivalent.



EXPLORATION 2 Simplifying Expressions with Rational Exponents

Work with a partner. Show that you can apply the properties of integer exponents to rational exponents by simplifying each expression. Use a calculator to check your answers.

a. $5^{2/3} \cdot 5^{4/3}$ **b.** $3^{1/5} \cdot 3^{4/5}$ **c.** $(4^{2/3})^3$

d. $(10^{1/2})^4$ **e.** $\frac{8^{5/2}}{8^{1/2}}$ **f.** $\frac{7^{2/3}}{7^{5/3}}$

EXPLORATION 3 Simplifying Products and Quotients of Radicals

Work with a partner. Use the properties of exponents to write each expression as a single radical. Then evaluate each expression. Use a calculator to check your answers.

a. $\sqrt{3} \cdot \sqrt{12}$ **b.** $\sqrt[3]{5} \cdot \sqrt[3]{25}$ **c.** $\sqrt[4]{27} \cdot \sqrt[4]{3}$

d. $\frac{\sqrt{98}}{\sqrt{2}}$ **e.** $\frac{\sqrt[4]{4}}{\sqrt[4]{1024}}$ **f.** $\frac{\sqrt[3]{625}}{\sqrt[3]{5}}$

Communicate Your Answer

4. How can you use properties of exponents to simplify products and quotients of radicals?

5. Simplify each expression.

a. $\sqrt{27} \cdot \sqrt{6}$ **b.** $\frac{\sqrt[3]{240}}{\sqrt[3]{15}}$ **c.** $(5^{1/2} \cdot 16^{1/4})^2$

5.2 Lesson

Core Vocabulary

simplest form of a radical, *p. 245*
conjugate, *p. 246*
like radicals, *p. 246*

Previous
properties of integer exponents
rationalizing the denominator
absolute value

What You Will Learn

- Use properties of rational exponents to simplify expressions with rational exponents.
- Use properties of radicals to simplify and write radical expressions in simplest form.

Properties of Rational Exponents

The properties of integer exponents that you previously learned can also be applied to rational exponents.

Core Concept

Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers, such that the quantities in each property are real numbers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
Power of a Product	$(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
Zero Exponent	$a^0 = 1, a \neq 0$	$213^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

COMMON ERROR

When you multiply powers, *do not* multiply the exponents. For example, $3^2 \cdot 3^5 \neq 3^{10}$.

EXAMPLE 1 Using Properties of Exponents

Use the properties of rational exponents to simplify each expression.

a. $7^{1/4} \cdot 7^{1/2} = 7^{(1/4+1/2)} = 7^{3/4}$

b. $(6^{1/2} \cdot 4^{1/3})^2 = (6^{1/2})^2 \cdot (4^{1/3})^2 = 6^{(1/2 \cdot 2)} \cdot 4^{(1/3 \cdot 2)} = 6^1 \cdot 4^{2/3} = 6 \cdot 4^{2/3}$

c. $(4^5 \cdot 3^5)^{-1/5} = [(4 \cdot 3)^5]^{-1/5} = (12^5)^{-1/5} = 12^{[5 \cdot (-1/5)]} = 12^{-1} = \frac{1}{12}$

d. $\frac{5}{5^{1/3}} = \frac{5^1}{5^{1/3}} = 5^{(1-1/3)} = 5^{2/3}$

e. $\left(\frac{42^{1/3}}{6^{1/3}}\right)^2 = \left[\left(\frac{42}{6}\right)^{1/3}\right]^2 = (7^{1/3})^2 = 7^{(1/3 \cdot 2)} = 7^{2/3}$

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Simplify the expression.

1. $2^{3/4} \cdot 2^{1/2}$

2. $\frac{3}{3^{1/4}}$

3. $\left(\frac{20^{1/2}}{5^{1/2}}\right)^3$

4. $(5^{1/3} \cdot 7^{1/4})^3$

Simplifying Radical Expressions

The Power of a Product and Power of a Quotient properties can be expressed using radical notation when $m = \frac{1}{n}$ for some integer n greater than 1.

Core Concept

Properties of Radicals

Let a and b be real numbers and let n be an integer greater than 1.

Property Name	Definition	Example
Product Property	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$
Quotient Property	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$	$\frac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\frac{162}{2}} = \sqrt[4]{81} = 3$

EXAMPLE 2 Using Properties of Radicals

Use the properties of radicals to simplify each expression.

a. $\sqrt[3]{12} \cdot \sqrt[3]{18} = \sqrt[3]{12 \cdot 18} = \sqrt[3]{216} = 6$ — Product Property of Radicals

b. $\frac{\sqrt[4]{80}}{\sqrt[4]{5}} = \sqrt[4]{\frac{80}{5}} = \sqrt[4]{16} = 2$ — Quotient Property of Radicals

An expression involving a radical with index n is in **simplest form** when these three conditions are met.

- No radicands have perfect nth powers as factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

To meet the last two conditions, rationalize the denominator by multiplying the expression by an appropriate form of 1 that eliminates the radical from the denominator.

EXAMPLE 3 Writing Radicals in Simplest Form

Write each expression in simplest form.

a. $\sqrt[3]{135}$

b. $\frac{\sqrt[5]{7}}{\sqrt[5]{8}}$

SOLUTION

a. $\sqrt[3]{135} = \sqrt[3]{27 \cdot 5}$ — Factor out perfect cube.

$= \sqrt[3]{27} \cdot \sqrt[3]{5}$ — Product Property of Radicals

$= 3\sqrt[3]{5}$ — Simplify.

b. $\frac{\sqrt[5]{7}}{\sqrt[5]{8}} = \frac{\sqrt[5]{7}}{\sqrt[5]{8}} \cdot \frac{\sqrt[5]{4}}{\sqrt[5]{4}}$ — Make the radicand in the denominator a perfect fifth power.

$= \frac{\sqrt[5]{28}}{\sqrt[5]{32}}$ — Product Property of Radicals

$= \frac{\sqrt[5]{28}}{2}$ — Simplify.

For a denominator that is a sum or difference involving square roots, multiply both the numerator and denominator by the **conjugate** of the denominator. The expressions

$$a\sqrt{b} + c\sqrt{d} \qquad \text{and} \qquad a\sqrt{b} - c\sqrt{d}$$

are conjugates of each other, where a, b, c, and d are rational numbers.

EXAMPLE 4 Writing a Radical Expression in Simplest Form

Write $\dfrac{1}{5 + \sqrt{3}}$ in simplest form.

SOLUTION

$\dfrac{1}{5+\sqrt{3}} = \dfrac{1}{5+\sqrt{3}} \cdot \dfrac{5-\sqrt{3}}{5-\sqrt{3}}$ The conjugate of $5 + \sqrt{3}$ is $5 - \sqrt{3}$.

$= \dfrac{1(5-\sqrt{3})}{5^2 - (\sqrt{3})^2}$ Sum and Difference Pattern

$= \dfrac{5-\sqrt{3}}{22}$ Simplify.

Radical expressions with the same index and radicand are **like radicals**. To add or subtract like radicals, use the Distributive Property.

EXAMPLE 5 Adding and Subtracting Like Radicals and Roots

Simplify each expression.

a. $\sqrt[4]{10} + 7\sqrt[4]{10}$ **b.** $2(8^{1/5}) + 10(8^{1/5})$ **c.** $\sqrt[3]{54} - \sqrt[3]{2}$

SOLUTION

a. $\sqrt[4]{10} + 7\sqrt[4]{10} = (1 + 7)\sqrt[4]{10} = 8\sqrt[4]{10}$

b. $2(8^{1/5}) + 10(8^{1/5}) = (2 + 10)(8^{1/5}) = 12(8^{1/5})$

c. $\sqrt[3]{54} - \sqrt[3]{2} = \sqrt[3]{27} \cdot \sqrt[3]{2} - \sqrt[3]{2} = 3\sqrt[3]{2} - \sqrt[3]{2} = (3 - 1)\sqrt[3]{2} = 2\sqrt[3]{2}$

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Simplify the expression.

5. $\sqrt[4]{27} \cdot \sqrt[4]{3}$ **6.** $\dfrac{\sqrt[3]{250}}{\sqrt[3]{2}}$ **7.** $\sqrt[3]{104}$ **8.** $\sqrt[5]{\dfrac{3}{4}}$

9. $\dfrac{3}{6 - \sqrt{2}}$ **10.** $7\sqrt[5]{12} - \sqrt[5]{12}$ **11.** $4(9^{2/3}) + 8(9^{2/3})$ **12.** $\sqrt[3]{5} + \sqrt[3]{40}$

The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be positive, negative, or zero, sometimes absolute value is needed when simplifying a variable expression.

	Rule	Example
When *n* is odd	$\sqrt[n]{x^n} = x$	$\sqrt[7]{5^7} = 5$ and $\sqrt[7]{(-5)^7} = -5$
When *n* is even	$\sqrt[n]{x^n} = \lvert x \rvert$	$\sqrt[4]{3^4} = 3$ and $\sqrt[4]{(-3)^4} = 3$

Absolute value is not needed when all variables are assumed to be positive.

EXAMPLE 6 Simplifying Variable Expressions

Simplify each expression.

a. $\sqrt[3]{64y^6}$ **b.** $\sqrt[4]{\dfrac{x^4}{y^8}}$

STUDY TIP

You do not need to take the absolute value of *y* because *y* is being squared.

SOLUTION

a. $\sqrt[3]{64y^6} = \sqrt[3]{4^3(y^2)^3} = \sqrt[3]{4^3} \cdot \sqrt[3]{(y^2)^3} = 4y^2$

b. $\sqrt[4]{\dfrac{x^4}{y^8}} = \dfrac{\sqrt[4]{x^4}}{\sqrt[4]{y^8}} = \dfrac{\sqrt[4]{x^4}}{\sqrt[4]{(y^2)^4}} = \dfrac{|x|}{y^2}$

EXAMPLE 7 Writing Variable Expressions in Simplest Form

Write each expression in simplest form. Assume all variables are positive.

a. $\sqrt[5]{4a^8b^{14}c^5}$ **b.** $\dfrac{x}{\sqrt[3]{y^8}}$ **c.** $\dfrac{14xy^{1/3}}{2x^{3/4}z^{-6}}$

COMMON ERROR

You must multiply both the numerator *and* denominator of the fraction by $\sqrt[3]{y}$ so that the value of the fraction does not change.

SOLUTION

a. $\sqrt[5]{4a^8b^{14}c^5} = \sqrt[5]{4a^5a^3b^{10}b^4c^5}$ Factor out perfect fifth powers.

$= \sqrt[5]{a^5b^{10}c^5} \cdot \sqrt[5]{4a^3b^4}$ Product Property of Radicals

$= ab^2c\sqrt[5]{4a^3b^4}$ Simplify.

b. $\dfrac{x}{\sqrt[3]{y^8}} = \dfrac{x}{\sqrt[3]{y^8}} \cdot \dfrac{\sqrt[3]{y}}{\sqrt[3]{y}}$ Make denominator a perfect cube.

$= \dfrac{x\sqrt[3]{y}}{\sqrt[3]{y^9}}$ Product Property of Radicals

$= \dfrac{x\sqrt[3]{y}}{y^3}$ Simplify.

c. $\dfrac{14xy^{1/3}}{2x^{3/4}z^{-6}} = 7x^{(1-3/4)}y^{1/3}z^{-(-6)} = 7x^{1/4}y^{1/3}z^6$

EXAMPLE 8 Adding and Subtracting Variable Expressions

Perform each indicated operation. Assume all variables are positive.

a. $5\sqrt{y} + 6\sqrt{y}$ **b.** $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$

SOLUTION

a. $5\sqrt{y} + 6\sqrt{y} = (5 + 6)\sqrt{y} = 11\sqrt{y}$

b. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2} = 12z\sqrt[3]{2z^2} - 3z\sqrt[3]{2z^2} = (12z - 3z)\sqrt[3]{2z^2} = 9z\sqrt[3]{2z^2}$

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Simplify the expression. Assume all variables are positive.

13. $\sqrt[3]{27q^9}$ **14.** $\sqrt[5]{\dfrac{x^{10}}{y^5}}$ **15.** $\dfrac{6xy^{3/4}}{3x^{1/2}y^{1/2}}$ **16.** $\sqrt{9w^5} - w\sqrt{w^3}$

5.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** How do you know when a radical expression is in simplest form?

2. **WHICH ONE DOESN'T BELONG?** Which radical expression does *not* belong with the other three? Explain your reasoning.

$\sqrt[3]{\frac{4}{5}}$	$2\sqrt{x}$	$\sqrt[4]{11}$	$3\sqrt[5]{9x}$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, use the properties of rational exponents to simplify the expression. *(See Example 1.)*

3. $(9^2)^{1/3}$
4. $(12^2)^{1/4}$
5. $\dfrac{6}{6^{1/4}}$
6. $\dfrac{7}{7^{1/3}}$
7. $\left(\dfrac{8^4}{10^4}\right)^{-1/4}$
8. $\left(\dfrac{9^3}{6^3}\right)^{-1/3}$
9. $(3^{-2/3} \cdot 3^{1/3})^{-1}$
10. $(5^{1/2} \cdot 5^{-3/2})^{-1/4}$
11. $\dfrac{2^{2/3} \cdot 16^{2/3}}{4^{2/3}}$
12. $\dfrac{49^{3/8} \cdot 49^{7/8}}{7^{5/4}}$

In Exercises 13–20, use the properties of radicals to simplify the expression. *(See Example 2.)*

13. $\sqrt{2} \cdot \sqrt{72}$
14. $\sqrt[3]{16} \cdot \sqrt[3]{32}$
15. $\sqrt[4]{6} \cdot \sqrt[4]{8}$
16. $\sqrt[4]{8} \cdot \sqrt[4]{8}$
17. $\dfrac{\sqrt[5]{486}}{\sqrt[5]{2}}$
18. $\dfrac{\sqrt{2}}{\sqrt{32}}$
19. $\dfrac{\sqrt[3]{6} \cdot \sqrt[3]{72}}{\sqrt[3]{2}}$
20. $\dfrac{\sqrt[3]{3} \cdot \sqrt[3]{18}}{\sqrt[6]{2} \cdot \sqrt[6]{2}}$

In Exercises 21–28, write the expression in simplest form. *(See Example 3.)*

21. $\sqrt[4]{567}$
22. $\sqrt[5]{288}$
23. $\dfrac{\sqrt[3]{5}}{\sqrt[3]{4}}$
24. $\dfrac{\sqrt[4]{4}}{\sqrt[4]{27}}$
25. $\sqrt{\dfrac{3}{8}}$
26. $\sqrt[3]{\dfrac{7}{4}}$
27. $\sqrt[3]{\dfrac{64}{49}}$
28. $\sqrt[4]{\dfrac{1296}{25}}$

In Exercises 29–36, write the expression in simplest form. *(See Example 4.)*

29. $\dfrac{1}{1 + \sqrt{3}}$
30. $\dfrac{1}{2 + \sqrt{5}}$
31. $\dfrac{5}{3 - \sqrt{2}}$
32. $\dfrac{11}{9 - \sqrt{6}}$
33. $\dfrac{9}{\sqrt{3} + \sqrt{7}}$
34. $\dfrac{2}{\sqrt{8} + \sqrt{7}}$
35. $\dfrac{\sqrt{6}}{\sqrt{3} - \sqrt{5}}$
36. $\dfrac{\sqrt{7}}{\sqrt{10} - \sqrt{2}}$

In Exercises 37–46, simplify the expression. *(See Example 5.)*

37. $9\sqrt[3]{11} + 3\sqrt[3]{11}$
38. $8\sqrt[6]{5} - 12\sqrt[6]{5}$
39. $3(11^{1/4}) + 9(11^{1/4})$
40. $13(8^{3/4}) - 4(8^{3/4})$
41. $5\sqrt{12} - 19\sqrt{3}$
42. $27\sqrt{6} + 7\sqrt{150}$
43. $\sqrt[5]{224} + 3\sqrt[5]{7}$
44. $7\sqrt[3]{2} - \sqrt[3]{128}$
45. $5(24^{1/3}) - 4(3^{1/3})$
46. $5^{1/4} + 6(405^{1/4})$

47. **ERROR ANALYSIS** Describe and correct the error in simplifying the expression.

$$\begin{aligned} 3\sqrt[3]{12} + 5\sqrt[3]{12} &= (3 + 5)\sqrt[3]{24} \\ &= 8\sqrt[3]{24} \\ &= 8\sqrt[3]{8 \cdot 3} \\ &= 8 \cdot 2\sqrt[3]{3} \\ &= 16\sqrt[3]{3} \end{aligned}$$

48. **MULTIPLE REPRESENTATIONS** Which radical expressions are like radicals?

Ⓐ $(5^{2/9})^{3/2}$ Ⓑ $\dfrac{5^3}{(\sqrt[3]{5})^8}$

Ⓒ $\sqrt[3]{625}$ Ⓓ $\sqrt[3]{5145} - \sqrt[3]{875}$

Ⓔ $\sqrt[3]{5} + 3\sqrt[3]{5}$ Ⓕ $7\sqrt[4]{80} - 2\sqrt[4]{405}$

In Exercises 49–54, simplify the expression. *(See Example 6.)*

49. $\sqrt[4]{81y^8}$ 50. $\sqrt[3]{64r^3t^6}$

51. $\sqrt[5]{\dfrac{m^{10}}{n^5}}$ 52. $\sqrt[4]{\dfrac{k^{16}}{16z^4}}$

53. $\sqrt[6]{\dfrac{g^6h}{h^7}}$ 54. $\sqrt[8]{\dfrac{n^{18}p^7}{n^2p^{-1}}}$

55. **ERROR ANALYSIS** Describe and correct the error in simplifying the expression.

$$\sqrt[6]{\frac{64h^{12}}{g^6}} = \frac{\sqrt[6]{64h^{12}}}{\sqrt[6]{g^6}} = \frac{\sqrt[6]{2^6 \cdot (h^2)^6}}{\sqrt[6]{g^6}} = \frac{2h^2}{g}$$

56. **OPEN-ENDED** Write two variable expressions involving radicals, one that needs absolute value in simplifying and one that does not need absolute value. Justify your answers.

In Exercises 57–64, write the expression in simplest form. Assume all variables are positive. *(See Example 7.)*

57. $\sqrt{81a^7b^{12}c^9}$ 58. $\sqrt[3]{125r^4s^9t^7}$

59. $\sqrt[5]{\dfrac{160m^6}{n^7}}$ 60. $\sqrt[4]{\dfrac{405x^3y^3}{5x^{-1}y}}$

61. $\dfrac{\sqrt[3]{w} \cdot \sqrt{w^5}}{\sqrt{25w^{16}}}$ 62. $\dfrac{\sqrt[4]{v^6}}{\sqrt[7]{v^5}}$

63. $\dfrac{18w^{1/3}v^{5/4}}{27w^{4/3}v^{1/2}}$ 64. $\dfrac{7x^{-3/4}y^{5/2}z^{-2/3}}{56x^{-1/2}y^{1/4}}$

In Exercises 65–70, perform the indicated operation. Assume all variables are positive. *(See Example 8.)*

65. $12\sqrt[3]{y} + 9\sqrt[3]{y}$

66. $11\sqrt{2z} - 5\sqrt{2z}$

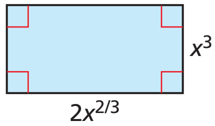
67. $3x^{7/2} - 5x^{7/2}$

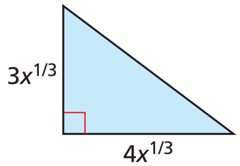
68. $7\sqrt[3]{m^7} + 3m^{7/3}$

69. $\sqrt[4]{16w^{10}} + 2w\sqrt[4]{w^6}$

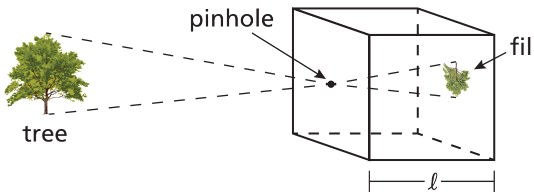
70. $(p^{1/2} \cdot p^{1/4}) - \sqrt[4]{16p^3}$

MATHEMATICAL CONNECTIONS **In Exercises 71 and 72, find simplified expressions for the perimeter and area of the given figure.**

71.


72.


73. **MODELING WITH MATHEMATICS** The optimum diameter d (in millimeters) of the pinhole in a pinhole camera can be modeled by $d = 1.9[(5.5 \times 10^{-4})\ell]^{1/2}$, where ℓ is the length (in millimeters) of the camera box. Find the optimum pinhole diameter for a camera box with a length of 10 centimeters.



74. **MODELING WITH MATHEMATICS** The surface area S (in square centimeters) of a mammal can be modeled by $S = km^{2/3}$, where m is the mass (in grams) of the mammal and k is a constant. The table shows the values of k for different mammals.

Mammal	Rabbit	Human	Bat
Value of k	9.75	11.0	57.5

a. Find the surface area of a bat whose mass is 32 grams.

b. Find the surface area of a rabbit whose mass is 3.4 kilograms (3.4×10^3 grams).

c. Find the surface area of a human whose mass is 59 kilograms.

75. **MAKING AN ARGUMENT** Your friend claims it is not possible to simplify the expression $7\sqrt{11} - 9\sqrt{44}$ because it does not contain like radicals. Is your friend correct? Explain your reasoning.

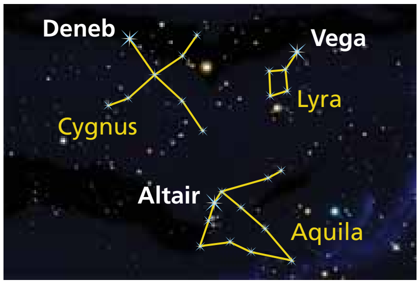
76. **PROBLEM SOLVING** The apparent magnitude of a star is a number that indicates how faint the star is in relation to other stars. The expression $\frac{2.512^{m_1}}{2.512^{m_2}}$ tells how many times fainter a star with apparent magnitude m_1 is than a star with apparent magnitude m_2.

Star	Apparent magnitude	Constellation
Vega	0.03	Lyra
Altair	0.77	Aquila
Deneb	1.25	Cygnus

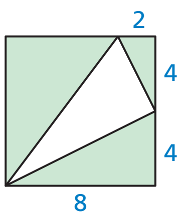
a. How many times fainter is Altair than Vega?

b. How many times fainter is Deneb than Altair?

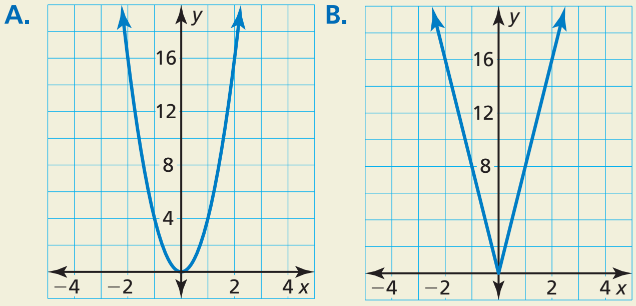
c. How many times fainter is Deneb than Vega?



77. **CRITICAL THINKING** Find a radical expression for the perimeter of the triangle inscribed in the square shown. Simplify the expression.



78. **HOW DO YOU SEE IT?** Without finding points, match the functions $f(x) = \sqrt{64x^2}$ and $g(x) = \sqrt[3]{64x^6}$ with their graphs. Explain your reasoning.



79. **REWRITING A FORMULA** You have filled two round balloons with water. One balloon contains twice as much water as the other balloon.

a. Solve the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$, for r.

b. Substitute the expression for r from part (a) into the formula for the surface area of a sphere, $S = 4\pi r^2$. Simplify to show that $S = (4\pi)^{1/3}(3V)^{2/3}$.

c. Compare the surface areas of the two water balloons using the formula in part (b).

80. **THOUGHT PROVOKING** Determine whether the expressions $(x^2)^{1/6}$ and $(x^{1/6})^2$ are equivalent for all values of x.

81. **DRAWING CONCLUSIONS** Substitute different combinations of odd and even positive integers for m and n in the expression $\sqrt[n]{x^m}$. When you cannot assume x is positive, explain when absolute value is needed in simplifying the expression.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Identify the focus, directrix, and axis of symmetry of the parabola. Then graph the equation. *(Section 2.3)*

82. $y = 2x^2$ **83.** $y^2 = -x$ **84.** $y^2 = 4x$

Write a rule for g. Describe the graph of g as a transformation of the graph of f. *(Section 4.7)*

85. $f(x) = x^4 - 3x^2 - 2x,\ g(x) = -f(x)$

86. $f(x) = x^3 - x,\ g(x) = f(x) - 3$

87. $f(x) = x^3 - 4,\ g(x) = f(x - 2)$

88. $f(x) = x^4 + 2x^3 - 4x^2,\ g(x) = f(2x)$