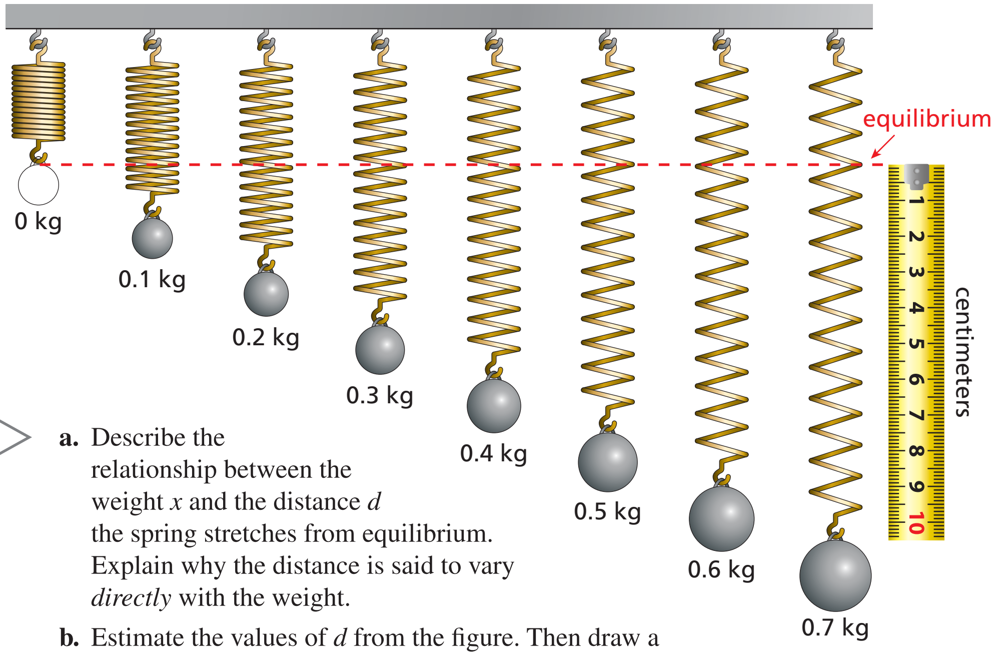
7.1 Inverse Variation

Essential Question

How can you recognize when two quantities vary directly or inversely?

EXPLORATION 1 Recognizing Direct Variation

Work with a partner. You hang different weights from the same spring.



REASONING QUANTITATIVELY

To be proficient in math, you need to make sense of quantities and their relationships in problem situations.

a. Describe the relationship between the weight x and the distance d the spring stretches from equilibrium. Explain why the distance is said to vary *directly* with the weight.

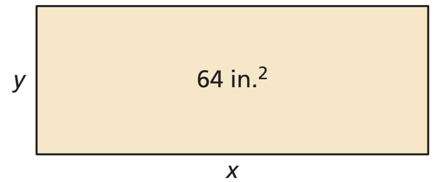
b. Estimate the values of d from the figure. Then draw a scatter plot of the data. What are the characteristics of the graph?

c. Write an equation that represents d as a function of x.

d. In physics, the relationship between d and x is described by *Hooke's Law*. How would you describe Hooke's Law?

EXPLORATION 2 Recognizing Inverse Variation

Work with a partner. The table shows the length x (in inches) and the width y (in inches) of a rectangle. The area of each rectangle is 64 square inches.



x	y
1	
2	
4	
8	
16	
32	
64	

a. Copy and complete the table.

b. Describe the relationship between x and y. Explain why y is said to vary *inversely* with x.

c. Draw a scatter plot of the data. What are the characteristics of the graph?

d. Write an equation that represents y as a function of x.

Communicate Your Answer

3. How can you recognize when two quantities vary directly or inversely?

4. Does the flapping rate of the wings of a bird vary directly or inversely with the length of its wings? Explain your reasoning.

7.1 Lesson

Core Vocabulary

inverse variation, *p. 360*
constant of variation, *p. 360*

Previous
direct variation
ratios

What You Will Learn

- Classify direct and inverse variation.
- Write inverse variation equations.

Classifying Direct and Inverse Variation

You have learned that two variables x and y show direct variation when $y = ax$ for some nonzero constant a. Another type of variation is called *inverse variation*.

Core Concept

Inverse Variation

Two variables x and y show **inverse variation** when they are related as follows:

$$y = \frac{a}{x}, \ a \neq 0$$

The constant a is the **constant of variation**, and y is said to *vary inversely* with x.

EXAMPLE 1 Classifying Equations

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

a. $xy = 5$

b. $y = x - 4$

c. $\frac{y}{2} = x$

SOLUTION

Given Equation	Solved for y	Type of Variation
a. $xy = 5$	$y = \frac{5}{x}$	inverse
b. $y = x - 4$	$y = x - 4$	neither
c. $\frac{y}{2} = x$	$y = 2x$	direct

STUDY TIP

The equation in part (b) does not show direct variation because $y = x - 4$ is not of the form $y = ax$.

Monitoring Progress

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Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

1. $6x = y$

2. $xy = -0.25$

3. $y + x = 10$

The general equation $y = ax$ for direct variation can be rewritten as $\frac{y}{x} = a$. So, a set of data pairs (x, y) shows direct variation when the ratios $\frac{y}{x}$ are constant.

The general equation $y = \frac{a}{x}$ for inverse variation can be rewritten as $xy = a$. So, a set of data pairs (x, y) shows inverse variation when the products xy are constant.

EXAMPLE 2 **Classifying Data**

Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

a.

x	2	4	6	8
y	-12	-6	-4	-3

b.

x	1	2	3	4
y	2	4	8	16

SOLUTION

a. Find the products xy and ratios $\frac{y}{x}$.

xy	-24	-24	-24	-24	The products are constant.
$\frac{y}{x}$	$\frac{-12}{2} = -6$	$\frac{-6}{4} = -\frac{3}{2}$	$\frac{-4}{6} = -\frac{2}{3}$	$-\frac{3}{8}$	The ratios are not constant.

▶ So, x and y show inverse variation.

ANALYZING RELATIONSHIPS

In Example 2(b), notice in the original table that as x increases by 1, y is multiplied by 2. So, the data in the table represent an exponential function.

b. Find the products xy and ratios $\frac{y}{x}$.

xy	2	8	24	64	The products are not constant.
$\frac{y}{x}$	$\frac{2}{1} = 2$	$\frac{4}{2} = 2$	$\frac{8}{3}$	$\frac{16}{4} = 4$	The ratios are not constant.

▶ So, x and y show neither direct nor inverse variation.

Monitoring Progress

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Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

4.

x	-4	-3	-2	-1
y	20	15	10	5

5.

x	1	2	3	4
y	60	30	20	15

Writing Inverse Variation Equations

EXAMPLE 3 **Writing an Inverse Variation Equation**

The variables x and y vary inversely, and $y = 4$ when $x = 3$. Write an equation that relates x and y. Then find y when $x = -2$.

ANOTHER WAY

Because x and y vary inversely, you also know that the products xy are constant. This product equals the constant of variation a. So, you can quickly determine that $a = xy = 3(4) = 12$.

SOLUTION

$y = \frac{a}{x}$ — Write general equation for inverse variation.

$4 = \frac{a}{3}$ — Substitute 4 for y and 3 for x.

$12 = a$ — Multiply each side by 3.

▶ The inverse variation equation is $y = \frac{12}{x}$. When $x = -2$, $y = \frac{12}{-2} = -6$.

EXAMPLE 4 Modeling with Mathematics

The time t (in hours) that it takes a group of volunteers to build a playground varies inversely with the number n of volunteers. It takes a group of 10 volunteers 8 hours to build the playground.

- Make a table showing the time that it would take to build the playground when the number of volunteers is 15, 20, 25, and 30.
- What happens to the time it takes to build the playground as the number of volunteers increases?

SOLUTION

1. **Understand the Problem** You are given a description of two quantities that vary inversely and one pair of data values. You are asked to create a table that gives additional data pairs.

2. **Make a Plan** Use the time that it takes 10 volunteers to build the playground to find the constant of variation. Then write an inverse variation equation and substitute for the different numbers of volunteers to find the corresponding times.

3. **Solve the Problem**

$t = \frac{a}{n}$ Write general equation for inverse variation.

$8 = \frac{a}{10}$ Substitute 8 for t and 10 for n.

$80 = a$ Multiply each side by 10.

The inverse variation equation is $t = \frac{80}{n}$. Make a table of values.

n	15	20	25	30
t	$\frac{80}{15} = 5$ h 20 min	$\frac{80}{20} = 4$ h	$\frac{80}{25} = 3$ h 12 min	$\frac{80}{30} = 2$ h 40 min

As the number of volunteers increases, the time it takes to build the playground decreases.

4. **Look Back** Because the time decreases as the number of volunteers increases, the time for 5 volunteers to build the playground should be greater than 8 hours.

$$t = \frac{80}{5} = 16 \text{ hours} \checkmark$$

LOOKING FOR A PATTERN

Notice that as the number of volunteers increases by 5, the time decreases by a lesser and lesser amount.

From $n = 15$ to $n = 20$, t decreases by 1 hour 20 minutes.

From $n = 20$ to $n = 25$, t decreases by 48 minutes.

From $n = 25$ to $n = 30$, t decreases by 32 minutes.

Monitoring Progress

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The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = 2$.

6. $x = 4, y = 5$ **7.** $x = 6, y = -1$ **8.** $x = \frac{1}{2}, y = 16$

9. **WHAT IF?** In Example 4, it takes a group of 10 volunteers 12 hours to build the playground. How long would it take a group of 15 volunteers?

7.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Explain how direct variation equations and inverse variation equations are different.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What is an inverse variation equation relating x and y with $a = 4$?	What is an equation for which the ratios $\frac{y}{x}$ are constant and $a = 4$?
What is an equation for which y varies inversely with x and $a = 4$?	What is an equation for which the products xy are constant and $a = 4$?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, tell whether x and y show *direct variation, inverse variation,* or *neither*. *(See Example 1.)*

3. $y = \frac{2}{x}$

4. $xy = 12$

5. $\frac{y}{x} = 8$

6. $4x = y$

7. $y = x + 4$

8. $x + y = 6$

9. $8y = x$

10. $xy = \frac{1}{5}$

In Exercises 11–14, tell whether x and y show *direct variation, inverse variation,* or *neither*. *(See Example 2.)*

11.

x	12	18	23	29	34
y	132	198	253	319	374

12.

x	1.5	2.5	4	7.5	10
y	13.5	22.5	36	67.5	90

13.

x	4	6	8	8.4	12
y	21	14	10.5	10	7

14.

x	4	5	6.2	7	11
y	16	11	10	9	6

In Exercises 15–22, the variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = 3$. *(See Example 3.)*

15. $x = 5, y = -4$

16. $x = 1, y = 9$

17. $x = -3, y = 8$

18. $x = 7, y = 2$

19. $x = \frac{3}{4}, y = 28$

20. $x = -4, y = -\frac{5}{4}$

21. $x = -12, y = -\frac{1}{6}$

22. $x = \frac{5}{3}, y = -7$

ERROR ANALYSIS **In Exercises 23 and 24, the variables x and y vary inversely. Describe and correct the error in writing an equation relating x and y.**

23. $x = 8, y = 5$

✗

$y = ax$

$5 = a(8)$

$\frac{5}{8} = a$

So, $y = \frac{5}{8}x$.

24. $x = 5, y = 2$

✗

$xy = a$

$5 \cdot 2 = a$

$10 = a$

So, $y = 10x$.

25. **MODELING WITH MATHEMATICS** The number y of songs that can be stored on an MP3 player varies inversely with the average size x of a song. A certain MP3 player can store 2500 songs when the average size of a song is 4 megabytes (MB). *(See Example 4.)*

   a. Make a table showing the numbers of songs that will fit on the MP3 player when the average size of a song is 2 MB, 2.5 MB, 3 MB, and 5 MB.

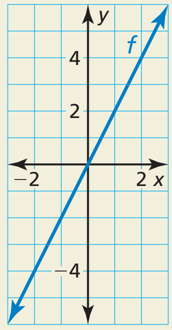
   b. What happens to the number of songs as the average song size increases?

26. **MODELING WITH MATHEMATICS** When you stand on snow, the average pressure P (in pounds per square inch) that you exert on the snow varies inversely with the total area A (in square inches) of the soles of your footwear. Suppose the pressure is 0.43 pound per square inch when you wear the snowshoes shown. Write an equation that gives P as a function of A. Then find the pressure when you wear the boots shown.

27. **PROBLEM SOLVING** Computer chips are etched onto silicon wafers. The table compares the area A (in square millimeters) of a computer chip with the number c of chips that can be obtained from a silicon wafer. Write a model that gives c as a function of A. Then predict the number of chips per wafer when the area of a chip is 81 square millimeters.

Area (mm^2), A	58	62	66	70
Number of chips, c	448	424	392	376

28. **HOW DO YOU SEE IT?** Does the graph of f represent inverse variation or direct variation? Explain your reasoning.



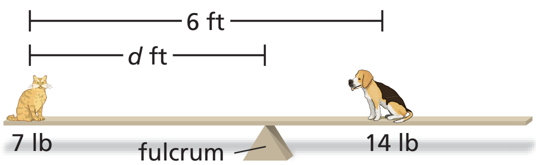
29. **MAKING AN ARGUMENT** You have enough money to buy 5 hats for \$10 each or 10 hats for \$5 each. Your friend says this situation represents inverse variation. Is your friend correct? Explain your reasoning.

30. **THOUGHT PROVOKING** The weight w (in pounds) of an object varies inversely with the square of the distance d (in miles) of the object from the center of Earth. At sea level (3978 miles from the center of the Earth), an astronaut weighs 210 pounds. How much does the astronaut weigh 200 miles above sea level?

31. **OPEN-ENDED** Describe a real-life situation that can be modeled by an inverse variation equation.

32. **CRITICAL THINKING** Suppose x varies inversely with y and y varies inversely with z. How does x vary with z? Justify your answer.

33. **USING STRUCTURE** To balance the board in the diagram, the distance (in feet) of each animal from the center of the board must vary inversely with its weight (in pounds). What is the distance of each animal from the fulcrum? Justify your answer.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Divide. *(Section 4.3)*

34. $(x^2 + 2x - 99) \div (x + 11)$

35. $(3x^4 - 13x^2 - x^3 + 6x - 30) \div (3x^2 - x + 5)$

Graph the function. Then state the domain and range. *(Section 6.4)*

36. $f(x) = 5^x + 4$

37. $g(x) = e^{x-1}$

38. $y = \ln 3x - 6$

39. $h(x) = 2 \ln (x + 9)$