1.3 Modeling with Linear Functions

COMMON CORE

Learning Standards
HSA-CED.A.2
HSF-IF.C.9
HSF-BF.A.1a
HSF-LE.A.2
HSS-ID.B.6a

Essential Question

How can you use a linear function to model and analyze a real-life situation?

EXPLORATION 1 Modeling with a Linear Function

Work with a partner. A company purchases a copier for $12,000. The spreadsheet shows how the copier depreciates over an 8-year period.

	A	B
1	**Year, *t***	**Value, *V***
2	0	$12,000
3	1	$10,750
4	2	$9,500
5	3	$8,250
6	4	$7,000
7	5	$5,750
8	6	$4,500
9	7	$3,250
10	8	$2,000
11		

a. Write a linear function to represent the value V of the copier as a function of the number t of years.

b. Sketch a graph of the function. Explain why this type of depreciation is called *straight line depreciation*.

c. Interpret the slope of the graph in the context of the problem.

MODELING WITH MATHEMATICS

To be proficient in math, you need to routinely interpret your results in the context of the situation.

EXPLORATION 2 Modeling with Linear Functions

Work with a partner. Match each description of the situation with its corresponding graph. Explain your reasoning.

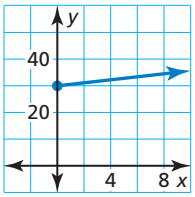
a. A person gives $20 per week to a friend to repay a $200 loan.

b. An employee receives $12.50 per hour plus $2 for each unit produced per hour.

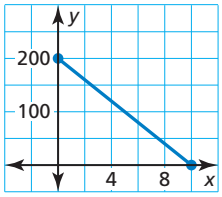
c. A sales representative receives $30 per day for food plus $0.565 for each mile driven.

d. A computer that was purchased for $750 depreciates $100 per year.

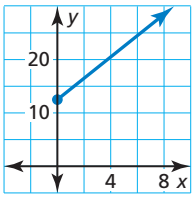
A.



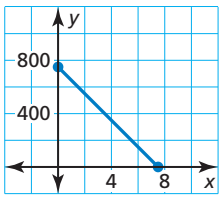
B.



C.



D.



Communicate Your Answer

3. How can you use a linear function to model and analyze a real-life situation?

4. Use the Internet or some other reference to find a real-life example of straight line depreciation.

a. Use a spreadsheet to show the depreciation.

b. Write a function that models the depreciation.

c. Sketch a graph of the function.

1.3 Lesson

What You Will Learn

- Write equations of linear functions using points and slopes.
- Find lines of fit and lines of best fit.

Core Vocabulary

line of fit, *p. 24*
line of best fit, *p. 25*
correlation coefficient, *p. 25*

Previous
slope
slope-intercept form
point-slope form
scatter plot

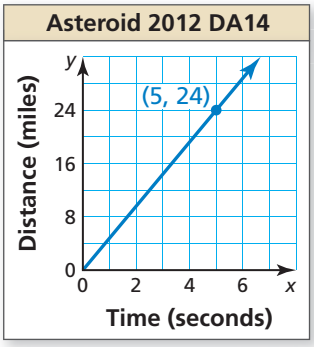
Writing Linear Equations

Core Concept

Writing an Equation of a Line

Given slope m and y-intercept b	Use slope-intercept form: $y = mx + b$
Given slope m and a point (x_1, y_1)	Use point-slope form: $y - y_1 = m(x - x_1)$
Given points (x_1, y_1) and (x_2, y_2)	First use the slope formula to find m. Then use point-slope form with either given point.

EXAMPLE 1 Writing a Linear Equation from a Graph



The graph shows the distance Asteroid 2012 DA14 travels in x seconds. Write an equation of the line and interpret the slope. The asteroid came within 17,200 miles of Earth in February, 2013. About how long does it take the asteroid to travel that distance?

SOLUTION

From the graph, you can see the slope is $m = \frac{24}{5} = 4.8$ and the y-intercept is $b = 0$. Use slope-intercept form to write an equation of the line.

$y = mx + b$ Slope-intercept form

$= 4.8x + 0$ Substitute 4.8 for m and 0 for b.

The equation is $y = 4.8x$. The slope indicates that the asteroid travels 4.8 miles per second. Use the equation to find how long it takes the asteroid to travel 17,200 miles.

$17{,}200 = 4.8x$ Substitute 17,200 for y.

$3583 \approx x$ Divide each side by 4.8.

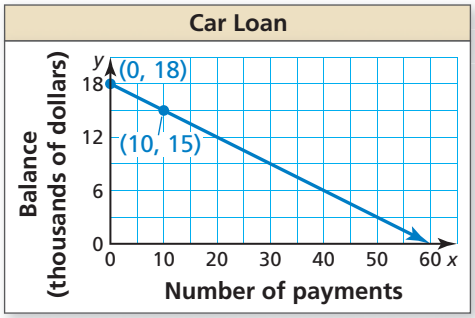
- Because there are 3600 seconds in 1 hour, it takes the asteroid about 1 hour to travel 17,200 miles.

REMEMBER
An equation of the form $y = mx$ indicates that x and y are in a proportional relationship.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

1. The graph shows the remaining balance y on a car loan after making x monthly payments. Write an equation of the line and interpret the slope and y-intercept. What is the remaining balance after 36 payments?



EXAMPLE 2 Modeling with Mathematics

Lakeside Inn	
Number of students, x	**Total cost, y**
100	\$1500
125	\$1800
150	\$2100
175	\$2400
200	\$2700

Two prom venues charge a rental fee plus a fee per student. The table shows the total costs for different numbers of students at Lakeside Inn. The total cost y (in dollars) for x students at Sunview Resort is represented by the equation

$$y = 10x + 600.$$

Which venue charges less per student? How many students must attend for the total costs to be the same?

SOLUTION

1. **Understand the Problem** You are given an equation that represents the total cost at one venue and a table of values showing total costs at another venue. You need to compare the costs.
2. **Make a Plan** Write an equation that models the total cost at Lakeside Inn. Then compare the slopes to determine which venue charges less per student. Finally, equate the cost expressions and solve to determine the number of students for which the total costs are equal.
3. **Solve the Problem** First find the slope using any two points from the table. Use $(x_1, y_1) = (100, 1500)$ and $(x_2, y_2) = (125, 1800)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1800 - 1500}{125 - 100} = \frac{300}{25} = 12$$

Write an equation that represents the total cost at Lakeside Inn using the slope of 12 and a point from the table. Use $(x_1, y_1) = (100, 1500)$.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 1500 = 12(x - 100)$	Substitute for m, x_1, and y_1.
$y - 1500 = 12x - 1200$	Distributive Property
$y = 12x + 300$	Add 1500 to each side.

Equate the cost expressions and solve.

$10x + 600 = 12x + 300$	Set cost expressions equal.
$300 = 2x$	Combine like terms.
$150 = x$	Divide each side by 2.

▶ Comparing the slopes of the equations, Sunview Resort charges \$10 per student, which is less than the \$12 per student that Lakeside Inn charges. The total costs are the same for 150 students.

4. **Look Back** Notice that the table shows the total cost for 150 students at Lakeside Inn is \$2100. To check that your solution is correct, verify that the total cost at Sunview Resort is also \$2100 for 150 students.

$y = 10(150) + 600$	Substitute 150 for x.
$= 2100$ ✓	Simplify.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

2. **WHAT IF?** Maple Ridge charges a rental fee plus a \$10 fee per student. The total cost is \$1900 for 140 students. Describe the number of students that must attend for the total cost at Maple Ridge to be less than the total costs at the other two venues. Use a graph to justify your answer.

Finding Lines of Fit and Lines of Best Fit

Data do not always show an *exact* linear relationship. When the data in a scatter plot show an approximately linear relationship, you can model the data with a **line of fit**.

Core Concept

Finding a Line of Fit

Step 1 Create a scatter plot of the data.

Step 2 Sketch the line that most closely appears to follow the trend given by the data points. There should be about as many points above the line as below it.

Step 3 Choose two points on the line and estimate the coordinates of each point. These points do not have to be original data points.

Step 4 Write an equation of the line that passes through the two points from Step 3. This equation is a model for the data.

EXAMPLE 3 Finding a Line of Fit

Femur length, x	Height, y
40	170
45	183
32	151
50	195
37	162
41	174
30	141
34	151
47	185
45	182

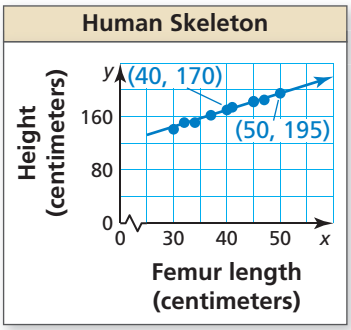
The table shows the femur lengths (in centimeters) and heights (in centimeters) of several people. Do the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate the height of a person whose femur is 35 centimeters long.

SOLUTION

Step 1 Create a scatter plot of the data. The data show a linear relationship.

Step 2 Sketch the line that most closely appears to fit the data. One possibility is shown.

Step 3 Choose two points on the line. For the line shown, you might choose (40, 170) and (50, 195).



Step 4 Write an equation of the line.

First, find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{195 - 170}{50 - 40} = \frac{25}{10} = 2.5$$

Use point-slope form to write an equation. Use $(x_1, y_1) = (40, 170)$.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 170 = 2.5(x - 40)$ Substitute for m, x_1, and y_1.

$y - 170 = 2.5x - 100$ Distributive Property

$y = 2.5x + 70$ Add 170 to each side.

Use the equation to estimate the height of the person.

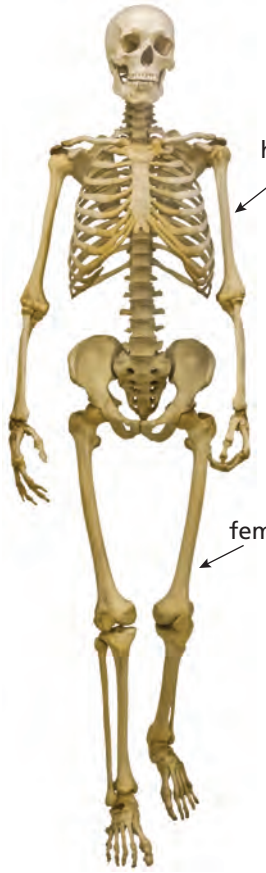
$y = 2.5(35) + 70$ Substitute 35 for x.

$= 157.5$ Simplify.

▶ The approximate height of a person with a 35-centimeter femur is 157.5 centimeters.

The **line of best fit** is the line that lies as close as possible to all of the data points. Many technology tools have a *linear regression* feature that you can use to find the line of best fit for a set of data.

The **correlation coefficient**, denoted by r, is a number from -1 to 1 that measures how well a line fits a set of data pairs (x, y). When r is near 1, the points lie close to a line with a positive slope. When r is near -1, the points lie close to a line with a negative slope. When r is near 0, the points do not lie close to any line.



EXAMPLE 4 Using a Graphing Calculator

Use the *linear regression* feature on a graphing calculator to find an equation of the line of best fit for the data in Example 3. Estimate the height of a person whose femur is 35 centimeters long. Compare this height to your estimate in Example 3.

SOLUTION

Step 1 Enter the data into two lists.

L1	L2	L3
40	170	------
45	183	
32	151	
50	195	
37	162	
41	174	
30	141	

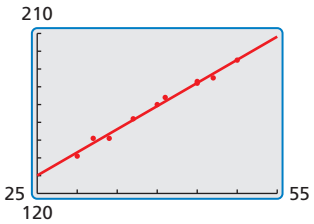
L1(1)=40

Step 2 Use the *linear regression* feature. The line of best fit is $y = 2.6x + 65$.

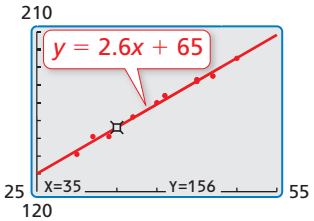
```
LinReg
 y=ax+b
 a=2.603570555
 b=64.99682074
 r²=.9890669473
 r=.9945184499
```

The value of r is close to 1.

Step 3 Graph the regression equation with the scatter plot.



Step 4 Use the *trace* feature to find the value of y when $x = 35$.



The approximate height of a person with a 35-centimeter femur is 156 centimeters. This is less than the estimate found in Example 3.

ATTENDING TO PRECISION

Be sure to analyze the data values to help you select an appropriate viewing window for your graph.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

3. The table shows the humerus lengths (in centimeters) and heights (in centimeters) of several females.

Humerus length, x	33	25	22	30	28	32	26	27
Height, y	166	142	130	154	152	159	141	145

a. Do the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate the height of a female whose humerus is 40 centimeters long.

b. Use the *linear regression* feature on a graphing calculator to find an equation of the line of best fit for the data. Estimate the height of a female whose humerus is 40 centimeters long. Compare this height to your estimate in part (a).

1.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

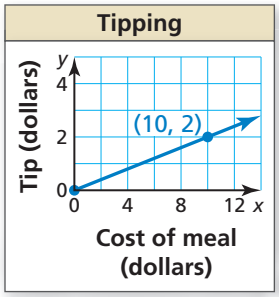
Vocabulary and Core Concept Check

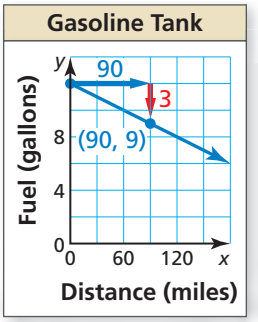
1. **COMPLETE THE SENTENCE** The linear equation $y = \frac{1}{2}x + 3$ is written in _____ form.

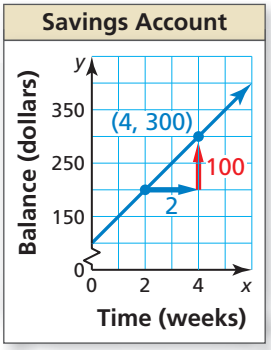
2. **VOCABULARY** A line of best fit has a correlation coefficient of -0.98. What can you conclude about the slope of the line?

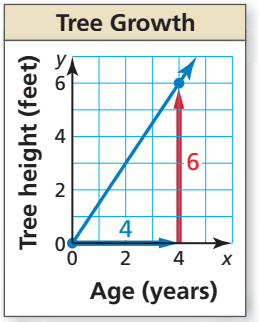
Monitoring Progress and Modeling with Mathematics

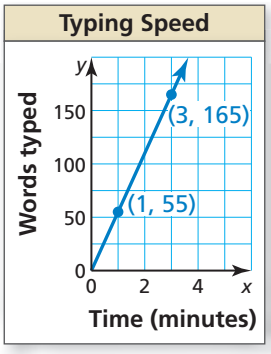
In Exercises 3–8, use the graph to write an equation of the line and interpret the slope. *(See Example 1.)*

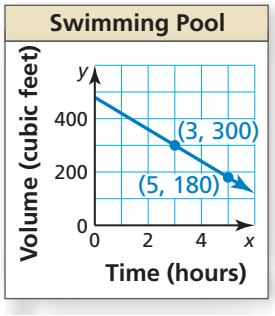
3. 

4. 

5. 

6. 

7. 

8. 

9. **MODELING WITH MATHEMATICS** Two newspapers charge a fee for placing an advertisement in their paper plus a fee based on the number of lines in the advertisement. The table shows the total costs for different length advertisements at the Daily Times. The total cost y (in dollars) for an advertisement that is x lines long at the Greenville Journal is represented by the equation $y = 2x + 20$. Which newspaper charges less per line? How many lines must be in an advertisement for the total costs to be the same? *(See Example 2.)*

Daily Times	
Number of lines, x	Total cost, y
4	27
5	30
6	33
7	36
8	39

10. **PROBLEM SOLVING** While on vacation in Canada, you notice that temperatures are reported in degrees Celsius. You know there is a linear relationship between Fahrenheit and Celsius, but you forget the formula. From science class, you remember the freezing point of water is 0°C or 32°F, and its boiling point is 100°C or 212°F.

   a. Write an equation that represents degrees Fahrenheit in terms of degrees Celsius.

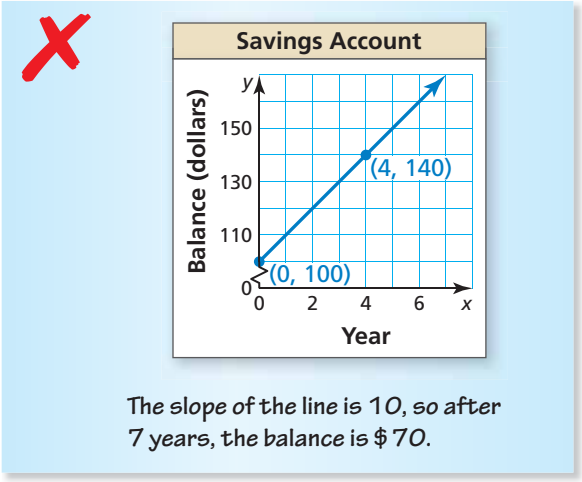
   b. The temperature outside is 22°C. What is this temperature in degrees Fahrenheit?

   c. Rewrite your equation in part (a) to represent degrees Celsius in terms of degrees Fahrenheit.

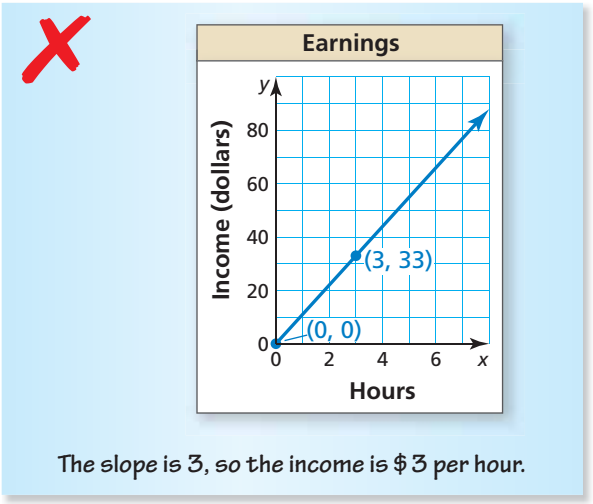
   d. The temperature of the hotel pool water is 83°F. What is this temperature in degrees Celsius?

ERROR ANALYSIS **In Exercises 11 and 12, describe and correct the error in interpreting the slope in the context of the situation.**

11.



12.



In Exercises 13–16, determine whether the data show a linear relationship. If so, write an equation of a line of fit. Estimate y when $x = 15$ and explain its meaning in the context of the situation. *(See Example 3.)*

13.

Minutes walking, x	1	6	11	13	16
Calories burned, y	6	27	50	56	70

14.

Months, x	9	13	18	22	23
Hair length (in.), y	3	5	7	10	11

15.

Hours, x	3	7	9	17	20
Battery life (%), y	86	61	50	26	0

16.

Shoe size, x	6	8	8.5	10	13
Heart rate (bpm), y	112	94	100	132	87

17. MODELING WITH MATHEMATICS The data pairs (x, y) represent the average annual tuition y (in dollars) for public colleges in the United States x years after 2005. Use the *linear regression* feature on a graphing calculator to find an equation of the line of best fit. Estimate the average annual tuition in 2020. Interpret the slope and y-intercept in this situation. *(See Example 4.)*

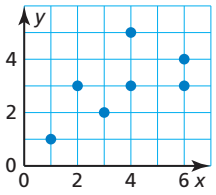
(0, 11,386), (1, 11,731), (2, 11,848)
(3, 12,375), (4, 12,804), (5, 13,297)

18. MODELING WITH MATHEMATICS The table shows the numbers of tickets sold for a concert when different prices are charged. Write an equation of a line of fit for the data. Does it seem reasonable to use your model to predict the number of tickets sold when the ticket price is $85? Explain.

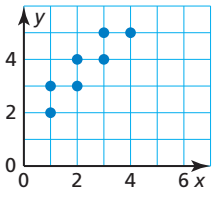
Ticket price (dollars), x	17	20	22	26
Tickets sold, y	450	423	400	395

USING TOOLS **In Exercises 19–24, use the *linear regression* feature on a graphing calculator to find an equation of the line of best fit for the data. Find and interpret the correlation coefficient.**

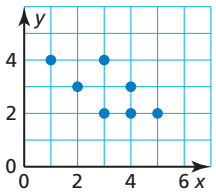
19.



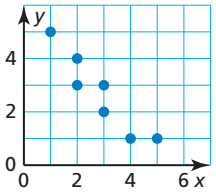
20.



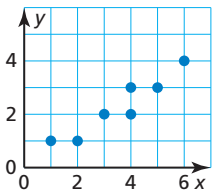
21.



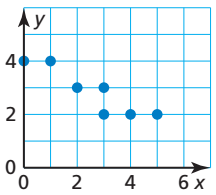
22.



23.

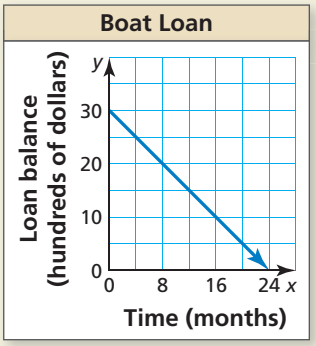


24.



25. OPEN-ENDED Give two real-life quantities that have (a) a positive correlation, (b) a negative correlation, and (c) approximately no correlation. Explain.

26. HOW DO YOU SEE IT? You secure an interest-free loan to purchase a boat. You agree to make equal monthly payments for the next two years. The graph shows the amount of money you still owe.



a. What is the slope of the line? What does the slope represent?

b. What is the domain and range of the function? What does each represent?

c. How much do you still owe after making payments for 12 months?

27. MAKING AN ARGUMENT A set of data pairs has a correlation coefficient $r = 0.3$. Your friend says that because the correlation coefficient is positive, it is logical to use the line of best fit to make predictions. Is your friend correct? Explain your reasoning.

28. THOUGHT PROVOKING Points A and B lie on the line $y = -x + 4$. Choose coordinates for points A, B, and C where point C is the same distance from point A as it is from point B. Write equations for the lines connecting points A and C and points B and C.

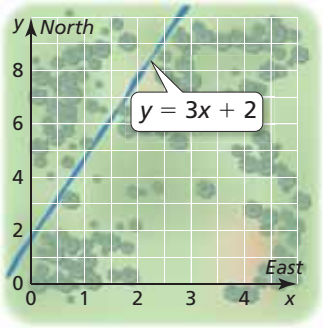
29. ABSTRACT REASONING If x and y have a positive correlation, and y and z have a negative correlation, then what can you conclude about the correlation between x and z? Explain.

30. MATHEMATICAL CONNECTIONS Which equation has a graph that is a line passing through the point $(8, -5)$ and is perpendicular to the graph of $y = -4x + 1$?

Ⓐ $y = \frac{1}{4}x - 5$ Ⓑ $y = -4x + 27$

Ⓒ $y = -\frac{1}{4}x - 7$ Ⓓ $y = \frac{1}{4}x - 7$

31. PROBLEM SOLVING You are participating in an orienteering competition. The diagram shows the position of a river that cuts through the woods. You are currently 2 miles east and 1 mile north of your starting point, the origin. What is the shortest distance you must travel to reach the river?



32. ANALYZING RELATIONSHIPS Data from North American countries show a positive correlation between the number of personal computers per capita and the average life expectancy in the country.

a. Does a positive correlation make sense in this situation? Explain.

b. Is it reasonable to conclude that giving residents of a country personal computers will lengthen their lives? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the system of linear equations in two variables by elimination or substitution. *(Skills Review Handbook)*

33. $3x + y = 7$
$-2x - y = 9$

34. $4x + 3y = 2$
$2x - 3y = 1$

35. $2x + 2y = 3$
$x = 4y - 1$

36. $y = 1 + x$
$2x + y = -2$

37. $\frac{1}{2}x + 4y = 4$
$2x - y = 1$

38. $y = x - 4$
$4x + y = 26$